a simple sensory system and a motor system enabling forward and rotational moves. The behavior itself, which involves moving in on any small object in the sensory field but 'standing clear' of any large object, seems rather straightforward. However, it turns out to be poorly learned by supervised methods. We explain this 'failure-to-learn' using a statistical analysis based on a qualitative distinction between two classes of generalization effect.

The paper is divided into six main sections. This, the first section, is an introduction. In the second section we describe the comparative study, the simulation setup used, the

the lowermost rectangle in the gure. To begin with only the larger of the two round objects exists.

set of examples. To obtain these, we repeatedly sampled the animat's reactions during simulation runs. This involved interrupting our simulation program in the middle of each time cycle and recording the sensory input received by the animat at that point, and the amount of drive being sent to the two wheels. The input/output pairs thus produced gave us the required training set.

The conditional-approach behavior entails producing three, basic behavioral responses to four scenarios. With no object appearing in the sensory field the animat must swivel rightwards by 10 degrees. With an object appearing at long-range, or a small object appearing at close-range the animat must execute a forwards move towards that object. (This might or might not involve a change in direction.) With a large object appearing at close-range the animat should remain stationary.

To ensure that each of these responses had an equal representation within the training data we used the following initialization regime. Each time the animat arrived at a small object or remained stationary for more than 20 cycles, we reinitialized the environment, changing the size of the single object, and randomly choosing a new position for it. Thus, in each successive phase of the simulation, the animat would be confronted by an object of a different size and different relative position. The sampled stimulus-responses pairs thus contained roughly equal numbers of the four responses.

Our general strategy for testing the efficiency of training (with a particular learning algorithm) was as follows. Following derivation and presentation of the relevant training set (see above) we would re-run the simulation program interrupting it in the middle of each cycle. The animat's current proximity inputs would then be presented as a 'test case' to the relevant learning algorithm. The output returned would be used to drive the wheels of the animat. At the end of the simulation run, we would evaluate the overall behavior as a reproduction of the desired behavior.

2.3 Format of training examples

The inputs from the sensory system were represented (for purposes of training) in the form of real numbers in the range 0.0-1.0. The inputs formed a normalized measure of proximity and embodied 10% noise. The amount of drive applied to the two wheels in each simulation step was represented in the form of two real numbers, also in the range 0.0-1.0. Thus, a full right turn with no forwards motion would appear in the training set as the pair $\langle 1.0, 0.0 \rangle$ (given the assumption that the first number sets the drive on the left wheel and the second number the drive on the right wheel).

A sample of training pairs derived for the conditional-approach task is shown

in Table 1. Note that the first seven numbers in each row (training pair) are the noisy proximity inputs. These are labeled $v1$, $v2$, $v3$ etc. The final two numbers in each row specify the required amount of drive to be applied to the two drive wheels. These are labeled d1 (amount of drive to the left wheel) and d2 (amount of drive to the right wheel). The first row shows a case of 'standing off' from a large object: the amount of drive for both wheels is 0.00 . The second row illustrates the default behavioral response (swivel ten degrees to the right) produced whenever all the proximity inputs are zeros (indicating no object has been sensed). The swivel effect is achieved by setting the amount of drive for the right wheel to be 0.3.

v1	v2	v3	v4	v5	v6	V ₇	d1	d2
0.00	0.00	0.00	0.27	0.38	0.33	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0 ₃	0.00
0.00	0.00	0.81	0.81	0.81	0.79	0.78	0.00	0.00
0.00	0.87	0.88	0.89	0.89	0.89	0.00	1.00	1.00
$\overline{0}.00$	0.00	0.00	$\overline{0}$ 27	0.38	0.33	0.00	0.00	0.00
0.73	0.74	0.00	0.00	0.00	0.00	0.00	0.4	1.00
0.81	0.81	0.81	0.79	0.78	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.85	0.84	0.82	0.00	1.00	0.80
0.00	0.00	0.00	0.78	0.78	0.00	0.00	1.00	1.00
0.00	0.00	0.00	0.00	0.76	0.77	0.76	1.00	0.80

Table 1:

2.4 Algorithms and parameter settings

The use of standard-format training sets enabled us to test the performance of any supervised learning algorithm on the conditional-approach problem. In practice we tested the performance of a wide range of algorithms including ID3 [4] and C4.5 [5], feed-forward network learning algorithms of the backpropagation family including `vanilla' backpropagation [6], a second-order method based on conjugate-gradient descent [7] and a second-order method based on Newton's method called `quickprop' [8]. We also tested a constructive network learning method called 'cascade-correlation' [8] and a classifier/genetic-algorithm combination based on Goldberg's 'simple classifier system' [9].

The standard ID3 algorithm has no user-definable parameters. Thus there is only one way to apply it to a particular training problem. It produces as output a standard-format decision tree in which the leaf nodes are labeled with specific output cases and each internal node tests the value of a particular input variable. $C4.5$ is a more efficient version of ID3 that enables various parameters to be set to control tree-pruning actions. However, in all cases reported we used the program in unrestricted mode, i.e., with parameters set so that it would perform no pruning whatsoever.

All the network algorithms tested operate by modifying the connection weights in a fixed, non-recurrent network of artificial neurons (using the standard logistic activation function). The efficiency of network learning is determined by feeding in novel inputs to the network and seeing what outputs are generated after the activation has propagated across all the relevant connections. When applying network learning algorithms the user must decide the internal architecture of the network3 and, in some cases, the learning and momentum rate. When testing the various network learning algorithms we experimented with a range of twolayered, feed-forward architectures (with complete inter-layer connectivity) but found that the best performance was obtained using nine hidden units; i.e. we settled on a 7-9-2 feed-forward architecture. All the results reported relate to this case.

When testing standard backpropagation we found that a learning rate of 0.5 and a momentum of 0.9 gave best results and these were the settings used in all the cases reported. When testing iterative learning algorithms (i.e., the network learning algorithms) we ran the algorithms for a minimum of 100,000 epochs of training (i.e., 100,000 complete sweeps through the entire training set).

In testing the classier-system/genetic algorithm combination we used an implementation based closely on Goldberg's 'simple classifier system'. The classifier population was configured to include a 50/50 mixture of intermediate and final classiers. That is to say, the actions for half the classiers in any population were output patterns and the actions for the other half were input patterns. The standard bucket-brigade algorithm was used with standard defaults (e.g., 11. 12 Tap . in results in results and the case of the company of the case of classier system, but that none of the algorithms provided satisfactory performance on this problem. In general, following training the animat would tend

duced by the nearest-neighbours algorithm (NN). This gure seems low but actually reveals relatively poor performance (for reasons explained above). The same goes for the other error rates shown. The columns headed `Meal freq.' and 'Nip freq.' show the 'meal' and 'nip' frequencies respectively for the various simulated animats.

 \ln

the leading animat (i.e., the total amount of drive that could be applied to the wheels) was arranged to be 125% that of the pursuing animat. Thus the leading animat had a small speed advantage over the pursuing animat. In Figure 5 we see a trace of a simulated animat producing the pursuit behavior. The pursuing animat is shown here using dashed lines. The leading animat is shown using unbroken lines.

Figure 5:

2.7 Results for obstacle-avoidance and pursuit

The results for this second phase of experiments can be summarized by saying that all the learning algorithms looked at were able to learn the two behaviors rather easily. The main performance figures are shown in Table 3. The column headed 'avoidance-CF' shows the crash frequencies for the various animats performing obstacle-avoidance while the column headed `pursuit-AD' shows the `average distance to target' for the animats executing the pursuit behavior. (The distances are proportional to the size of the space.) The row labels are the names of the relevant learning algorithms as before. Note that the crash frequencies and the average distances for all the trained animats are low when compared with the randomly moving animat.

In this dataset we can observe a number of instantiation n-tuples, henceforth called cases. First-order cases are instantiation 1-tuples. Examples include <x1=3>, <x3=0> and <x2=2>. Second-order cases are instantiation 2-tuples. An example is <x1=3, x2=1>. This case is observed in the fourth line of the training set. A second-order case from the second-order case from the second-order case \mathcal{A} $\langle x3=0, x1=2 \rangle$. Since there are only three variables in all there is exactly one

Case	Freq.
	1
$x2=2$	0.5
$x2=1$	0.5
$x3=1$	0.5
$x3=0$	0.5
$x1=3$	0.33
$x1=2$	0.33
$x1=1$	0.33
$x2=2 + x3=1$	0.33
$x2=1 + x3=0$	0.33
$x1=3 + x2=2$	0.17
$x1=2 + x2=2$	0.17
$x1=1 + x2=2$	0.17
$x1=3 + x2=1$	0.17
$x1=3 + x3=1$	0.17
$x1=2 + x2=1$	0.17
$x1=2 + x3=1$	$0.\overline{17}$
$x2=1 + x3=1$	$0.17\,$
$\overline{x1=3 + x3=0}$	0.17
$x1=1 + x3=1$	$0.17\,$

Table 4:

3.2 Type-1 versus type-2 frequencies

A clear distinction must be made between cases (such as those considered above) that can be observed directly in the training data, and cases that can only be observed indirectly. For our purposes,

- Type-1 regularity: divergence from chance-levels in type-1 frequencies.
- Type-2 regularity: divergence from chance-levels in type-2 frequencies.

To place this in a concrete setting, consider the example training sets shown above. The output variable x3 is a binary variable. Thus the frequency for either of its two possible instantiations is exactly 0.5. When we look at the type-1 conditional frequencies for the training data we see that most of the values are at or close to their chance-level of 0.5. However, when we derive relevant the type-2 conditional frequencies (after recoding in the suggested way) we obtain a frequency table in which *every* value diverges *maximally* from its chance level. Intuitively, then, the training set can be classied as exhibiting more type-2 regularity than type-1.⁶

The frequency effects brought to light by the recoding translate naturally into a completely general input/output rule. The table of type-2 conditional frequencies makes it obvious that $x3=1$ if and only if $x4=1$. From this we trivially obtain the input/output rule 'x3=1 if x4=1; otherwise x3=0.' Thus we see how the recoding effectively brings the regularity underlying the training set to the surface. Once this has happened it is a straightforward matter for a learning algorithm to exploit it. Recognizing the strong, mutual interdependence between learning and regularity leads us to distinguish three classes of learning problem.

- Pure type-1 learning problems: problems that involve exploiting type-1 regularities only,
- Pure type-2 learning problems: problems that involve exploiting type-2 regularities only, and
- Hybrid problems: problems that involve exploiting some mixture of

Every single first and second-order conditional frequency for this mapping (for values of the output variable x4) is at its chance level of 0.5. And, in fact, the frequency statistics for parity mappings are always like this. If we are dealing with n-bit parity then the highest order, non-degenerate frequencies are the $(n-1)$ th-order frequencies. Given binary variables we will necessarily find exactly two occurrences of each (n-1)th-order case in the training set, and these two cases will necessarily show a different value for the 'other' variable. Thus the conditional frequencies for the case in

the basis of an input/output rule that implicitly invokes a reformulation step) may well exhibit 'spurious' type-1 regularity.

The example training set used above illustrates this. The problem is 'intrinsically type-2' since the input/output rule used to construct the pairs assumes the reformulation step of converting the original input variables to their difference. And yet the type-1 frequencies show some marked, non-chance values (see the frequencies for the cases $\langle x2=1 \rangle$ and $\langle x2=2 \rangle$). These would be straightforwardly exploited by an algorithm such as Perceptron [14] or ID3 [4].

Even where intrinsically type-2 problems show very little spurious type-1 regularity they may still be solved by sophisticated learning algorithms such as packpropagation [6], cascade-correlation [8] or copycat [15]. Tt is, of course, well known that backpropagation can solve problems based on parity, symmetry or `shift' relationships and all these typically involve the algorithm deriving what can be thought of as an internal recoding scheme.

However, we should not over-estimate the generality of such methods. All of them introduce restrictive assumptions about the nature of the type-2 regularity to be discovered. Backpropagation for example effectively assumes that the required reformulation can be expressed in terms of the user-defined architecture of semi-linear transfer functions, and that it can be discovered by the gradient descent method embodied in the learning algorithm. If the assumption is invalid, the learning necessarily fails.

This may help to explain why backpropagation usually *fails* to solve low-order parity problems when they are presented as generalization problems (i.e., when some cases are held back for testing purposes). The graph shown in Figure 7 was produced from an empirical survey that involved running backpropagation on 4-bit parity generalization problems (with four, randomly selected cases used as unseens) using a wide range of internal architectures, including the theoretical minimal architecture. All the curves in the upper half of the graph are error profiles for the testing set of four cases. All the curves in the lower half of the graph are error profiles for the training set. There are 32 pairs of curves in all although many of them are bunched together in two clumps at the far left of the graph. Rather obviously, generalization over the testing cases was never observed to improve much beyond the chance level in any of the runs recorded. But the point to note is that the training-set error proles typically go to zero rather rapidly. This tells us that the generalization failure occurs in the context of perfectly successful learning, i.e., perfect acquisition of the training cases. This is a particularly concrete sort of generalization failure since it cannot be

⁸This latter is not usually presented as a learning algorithm. However it can certainly be construed as such.

 9 The error measure is the average difference between actual and target activations. For these experiments we used standard learning parameters; i.e., a learning rate of 0.5 and a momentum of 0.9.

and pursuit) is actually based on a type-2 rather than a type-1 regularity. But do we have any good reason to believe this is the case?

Type-2 problems are based on relativistic rules 4.1

As it turns out, there is a way of measuring the degree of type-1 regularity in a given training set and we can apply this measure to show that simulation-derived training sets for conditional-approach do embody low levels of type-1 regularity when compared against training sets for the other two behaviors. However, before describing the measure let us first make some informal comments about the nature of the two regularity classes.

A (supervised) learning problem is always dened in terms of a target input/output mapping. In all reasonable problems, the mapping is based on an input/output 'rule' of some sort and it is the task of the learning to discover this rule and represent it in such a way as to enable unseen inputs to be mapped onto their correct outputs. In the simplest case, the rule refers (perhaps implicitly) to particular input-variable values. If it does so, we will expect to see correlations between particular input values and particular output values. In other words, the rule will tend to have a 'representation' in the form of strong, type-1, conditional-frequency effects.

But of course the rule may not refer $-$ even implicitly $-$ to particular input values. It may refer to relationships between input values. The parity rule is an obvious example. The parity rule does not 'care' about explicit input values. It only cares whether there is a particular relationship among them. And in this case, where the rule only takes account of input-value relationships, it will not have the effect of producing type-1 correlations in the training set. The rule has

the apparent width and the apparent closeness of the object.10 The rules for obstacle-avoidance and pursuit, on the other hand, are both based on establishing a direct correspondence between the apparent closeness of an object in the sensory field and a particular behavioral response. In the case of pursuit, the correspondence is a simple matter of sensory stimulation being transformed into drive-wheel activity: objects appearing in particular parts of the sensory field cause particular amounts of drive to be applied to the wheels to ensure an appropriate move/turn. In the case of obstacle-avoidance the correspondence is a matter of sensory stimulation indicative of very near objects inhibiting drivewheel altogether. Thus in both cases we are dealing with a non-relativistic rule; i.e., a rule that takes account of explicit values rather than relationships between them.

4.2 Measuring type-1 regularity

We find ourselves, then, increasingly in favour of the idea that the relative hardness of 'conditional-approach' is due to the fact that this problem is based on a type-2 rather than a type-1 effect. However, to be more certain of this conclusion we should attempt to measure the amount of type-1 regularity in the various training sets and show that conditional-approach does indeed differ in this respect. (Measuring type-2 regularity is out of the question since it would involve implicitly searching the whole of Turing-machine space.)

A measure of type-1 regularity must satisfy certain constraints. Obviously, it must be sensitive to the degree to which type-1 frequencies diverge from chance levels (since this is the essence of the effect in question) but it must not be insensitive to dependencies between these effects. In almost all cases, we will have many frequency effects associated with the same aspect of the regularity. Thus if we simply work out the overall divergence from chance-frequencies (e.g., as an average or total) we will compute a value which overstates the case.

To get over this problem we must measure divergence within a subset of independent frequency effects. One way to obtain such a set involves using Bayesian inference as an output-generation process. The procedure is as follows. First of all we compute all empirical frequencies. Then, we find the smallest subset of frequencies that $-$ when treated as conditional probabilities $-$ successfully generate (via Bayesian inference) correct outputs for all inputs. This opera-

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given inputs.¹¹ We select a given input, we work out which cases it exhibits, and we then integrate the relevant conditional probabilities to derive probability distributions for the output values.

Finding the smallest subset of frequencies (probabilities) that completely 'captures' the training set (i.e., enables correct outputs to be generated for all inputs) is guaranteed to give us the set we want. Since the set is of a minimal size, we know that it must minimize (even if it does not abolish) the overall dependence between the effects. (If it did not, it would be possible to shuffle some cases in and some cases out to achieve a smaller set.) Since it captures the entire training set, we know that it cannot exclude any effect that reflects an aspect of the input/output rule.

Having derived a minimal subset of maximally independent frequency effects we still have the problem of measuring the overall divergence from chance-levels. It is not clear what tradeoff should apply between effects which diverge strongly from chance levels, and effects which diverge weakly but are nevertheless more general (i.e., useful) with respect to the capture of the training set. We can, however, finesse this problem entirely by observing that the relative size of the subset, relative to the original training set, will itself provide a satisfactory measure of the level of type-1 regularity. To compute the relative size we find the ratio between the number of variable instantiations used in the training set, and the number used in the frequency subset. With high levels of type-1 regularity we expect to see stronger and the more independent frequency effects. When such effects exist we will need fewer of them for a complete capture of the training set. Thus the relative size of the smallest subset that completely captures the training set measures the overall level of type-1 regularity.

A natural way to summarize this measure of type-1 regularity is as a compression ratio [16]. The compression ratio we define as the ratio between the number of variable instantiations used in specifying the frequency effects and the number used in the original training set. We then define the *empirical redundancy* of a particular training set to be the compression ratio that is achieved when we find the minimal subset of empirical frequency effects that completely captures the training set.

We can show how the measure works by applying it to the toy learning problem described above. This problem involves producing a 1 if the difference between the two input variables is 1. Our initial formulation of the problem contained just six cases and if we apply the measure to that training set we obtain a value that is strongly biased by the overhead costs of frequency-subset specification. However, we can derive a more realistic (i.e., larger) training set for the problem by allowing the input values to vary between 0 and 4. This gives us the training pairs shown on the left below.

 11 Of course, doing so entails assuming the statistical independence of variables.

The empirical redundancy of this training set turns out to be 14.6%. Reconfiguring the training pairs, adding in an extra input variable whose value is just the difference between the first two input variables, gives us a training set whose empirical redundancy is 89%, i.e., a great deal higher. The difference between these values clearly reflects what we already know to be the case: that the derived pairs effectively reify (in type-1 form) the type-2 regularity in the original pairs.12

When we apply our measure of type-1 regularity to simulation-derived training sets for the three learning problems we find $\overline{}$ as we expected $\overline{}$ that the value for conditional-approach stands out as a special case.¹³ When we tested training

¹²The discrepancy between the two redundancies is particularly extreme due to the fact that in this case we added the extra variable rather than substituted it for the original two input variables.

 $13\,\text{In}$ computing these measures we adopted a probabilistic approach to case identity. In the conventional scenario, case identity is a clear-cut issue: any two cases either are or are

sets for pursuit, obstacle-avoidance and conditional-approach we found that the empirical redundancy of the latter was markedly lower, see Table 8. It is not quite as low a value as we see in the case of the toy example above. However, this is only to be expected since with a much larger training set and many more input variables we expect to see many more spurious type-1 regularities. The hypothesis regarding the type-2 nature of conditional-approach, then, seems to be borne out by this particular statistical analysis.

Behavior	Empirical redundancy
Obstacle-avoidance	86.6
Pursuit	931
Conditional-approach	63 3

Table 8:

⁵ How could conditional-approach be learned?

Given the fact that none of the learning algorithms tested seemed able to deal with conditional-approach, it is natural to ask what sort of algorithm is actually required for this problem. A

of the conditional-approach problem. Thus any one of our learning algorithms, provided with an appropriate recoding ability, should be able to solve the problem. In the case of classical (i.e. program-based) learning algorithms it is easy to imagine how the recoding might be achieved: it would simply involve an implementation of the steps for the relevant control procedure. In the case of a network-based learning the question of the implementation of the recoding is less obvious. The results with respect to the various network learning algorithms suggest that a single-net implementation of the behavior may be difficult to obtain. However, it turns out to be fairly straightforward to `hand-code' a multi-network implementation of the behavior.

As we have noted, at the most basic level, conditional-approach is all about performing a size-discrimination given only range and location information. The size of an object in the sensory field is a relativistic property of the apparent closeness and the apparent width of the object. As objects get closer they appear to get wider. Thus the actual width of an object is a function of the ratio between the apparent width and the apparent closeness. An obvious decomposition of the size-discrimination task, then, involves computing the actual size of the object by finding the ratio between the apparent closeness and the apparent width.

We can build a subnet for that implements this decomposition as follows. The task of calculating the ratio can be roughly approximated by feeding activation values representing apparent closeness and apparent width into a unit with a bipolar activation function (e.g., tanh). If one connection is positively weighted and one negatively weighted the activation of the sigmoid unit will be 'flat' (i.e., zero) just in case the ratio between the two values is close to 1. By arranging for any activation, positive or negative at this unit to inhibit some other unit, which is biased on, we can effectively obtain a subnet that provides a feature detector for the case in which the ratio between the two values is at some particular level

need only establish positive connections from all these hidden units to a single, linear output unit. The activation of the output unit will effectively measure the number of activated hidden units and therefore of the size of the object in the sensory field.

This decomposition solves the problem of building a largeness detector. However, for a complete solution we need a network that integrates this detector into a more general structure that also serves to implement the various behavioral responses. A plausible approach might involve building an `approach net' that could potentially serve the dual purpose of (1) implementing the basic approach operation and (2) carrying out the computation of apparent width. A possible architecture for this subnet is shown in Figure 8. The network has seven input units via which the seven proximity stimuli are received. Each of these inputs feeds directly on into a single hidden unit whose bias and threshold is set so as to achieve the zero-thresholding effect described above. Activation from these hidden units then flows on into the two, main output units. Each of these controls the drive applied to one of the wheels, with the amount of drive corresponding linearly to the amount of activation.

Figure 8:

The weights between the hidden units to the two main output units are arranged

input unit, the

6 Summary and concluding comments

The paper sought to investigate the extent to which learning and evolutionary methods can be used to obtain simple, adaptive behaviors. It presented the results of a comparative study that looked at the conditional-approach behavior. The results of the study showed that several, powerful learning methods are unable to successfully learn the conditional-approach behavior even though they are perfectly capable of learning other, closely related behaviors such as obstacleavoidance and pursuit.

The failure on conditional-approach training was explained using a statistical analysis. This showed that learning problems may involve discovering some blend of two types of regularity and that problems that primarily involve discovering the more accessible (type-1) form of regularity are likely to be more easily solved in general. Formal and informal arguments were put forward to establish that the conditional-approach learning problem involves exploiting the less accessible, type-2 form of regularity. The implication was then drawn out

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